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Electroweak Radiative Corrections

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ABSTRACT

A new framework to study electroweak physics at one-loop level in general $SU(2)_L \times U(1)_Y$ theories is introduced. It separates the 1-loop corrections into two pieces: process specific ones from vertex and box contributions and the universal ones due to contributions to the gauge boson propagators. The latter are parametrized in terms of four effective form factors, $\bar{e}^2(q^2)$, $\bar{s}^2(q^2)$, $\bar{g}_Z^2(q^2)$, and $\bar{g}_W^2(q^2)$, correspondingly to $\gamma\gamma$, γZ , ZZ , and WW propagators. By assuming only the standard model contributions to the process specific corrections, the magnitudes of the four form factors are determined at $q^2 = 0$ and at $q^2 = m_Z^2$ from all available precision experiments. These values are then compared systematically with the predictions of the $SU(2)_L \times U(1)_Y$ theories. No deviation from the standard model has been identified. Plausible range of the top quark mass is then obtained for a given Higgs boson mass and α_s .

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1. PRECISION EXPERIMENTS CONFRONT SUSY-GUT

One of the most exciting developments of recent years has been the observation [1] that the electroweak mixing angle $\sin^2 \theta_W$ measured precisely at LEP agrees excellently with the prediction of the supersymmetric (SUSY) $SU(5)$ grand unification theory (GUT). The agreement is so impressive that we can hope in the near future to learn about SUSY particle masses [2, 3] with a better measurement of the QCD coupling constant. It has been argued [4] that the uncertainty in the GUT scale particle masses screens any possible effects of SUSY particle threshold corrections to the coupling constant unification condition. The works of refs. [2, 3] showed that the non-observation of proton decay effectively constrains the GUT particle contributions to the coupling constant unification and that our hope of learning about the SUSY mass scale from the precision measurements has been revived.

In fact we can tell more about these new particles, about their quantum numbers, if this grand unification of the three couplings is not merely an accident. This is because the old $SU(5)$ model, the grand unification model without new particles, predicts too small a value of $\sin^2 \theta_W$ (~ 0.21) and at the same time too rapid proton decay ($\tau_p \sim 10^{29} years$), which are both clearly inconsistent with experiments. It is instructive to note that the prediction for $\sin^2 \theta_W$ can change only by the introduction of an incomplete $SU(5)$

multiplet; the only example in the SM other than the gauge bosons themselves is the Higgs doublet, whose triplet partner of the SU(5) 5-plet should be super-heavy. A simple exercise shows that we can reproduce roughly the observed values of $\sin^2 \theta_W$ (~ 0.23) by simply introducing five more Higgs doublets to the SM. This, however, necessarily leads to further shortening of the proton lifetime since the unification scale ($\sim m_X$) decreases significantly as we introduce more Higgs doublets. We can intuitively understand this trend because the Higgs bosons make the SU(2) and U(1) couplings larger at high energies, while the SU(3) coupling remains unaffected by them. The point at which the three couplings meet should hence be lower in the energy scale. In order to raise the unification point to avoid rapid proton decay, we should also make the SU(3) coupling large at high energies. We therefore need new colored particles as well at the electroweak scale, such as scalar gluons or leptoquarks [5]. It is instructive to note that the six Higgs doublets as found above are exactly what the minimal SUSY-SM effectively predicts, with its two Higgs doublets each accompanied by fermionic partners, because each fermionic partner of the scalar doublet contributes twice as much to the running of the gauge coupling constants. Furthermore, the colored particles that are necessary to enlarge the proton lifetime are supplied in this model as gluinos and squarks. We observe here that the minimal SUSY-SM, and hence the SUSY-SU(5) model, very naturally satisfies the two experimental requirements of the coupling constant unification.

What is exciting about this exercise is that there now seems to be a strong indication that new particles and new interactions among them may exist at the electroweak scale. They may be produced at the Tevatron, LEP2 and at super colliders. Their effects could be observed in precision experiments through quantum corrections prior to their discovery. The effects can be significant if some of the new particles are as light as weak bosons or if there exist new strong interactions among them. Even in the absence of such a signal, we will learn more about possible new particle spectra from the future precision experiments. This is our motivation to study electroweak radiative corrections and we developed recently a new approach that allows us to look for new physics effects systematically [6]. We hope that the crucial roles of the future precision experiments at both high and low energies are made clear in this new framework.

2. A NEW FRAMEWORK FOR 1-LOOP ELECTROWEAK PHYSICS

Since what we want to learn from the electroweak precision experiments are the possible effects of new physics beyond the SM, whose exact nature is unknown, we would like to analyse the data in a framework which allows interpretations in wider classes of theoretical models. On the other hand the framework cannot be too general, since our ability to identifying effects of new physics from the precision experiments relies on the renormalizability of the electroweak theory which allows us to predict many observables in terms of a few parameters up to finite quantum corrections. Because the SM corrections are precisely known, those experiments which are sensitive to the quantum effects have a chance to identify a signal of physics beyond the SM. We therefore restrict ourselves to models that respect $SU(2)_L \times U(1)_Y$ gauge symmetry which breaks spontaneously down to $U(1)_{EM}$. In our approach, all new physics contributions that do not respect the spontaneously broken $SU(2)_L \times U(1)_Y$ gauge symmetry can be identified by our inability to fit the data successfully within our framework: these exotic interactions include all non-

renormalizable effective interactions among light quarks and leptons that may arise from an exchange of a heavy particle such as a new gauge boson or leptoquark boson, or from new strong interactions that bind common constituents of quarks and leptons.

Our restriction on the electroweak gauge group implies in the tree level that all quarks and leptons couple to the electroweak gauge bosons universally with the same coupling constant as long as they have common electroweak quantum numbers. This universality of the gauge boson coupling to quarks and leptons can in general be violated at the quantum level. It has widely been recognized, however, that this universality of the couplings holds true even in the one-loop level in a wider class of models where new particles affect the precision experiments only via their effects on the electroweak gauge boson propagators [7–14]. This class of effects due to new physics is often called oblique [7, 11] or propagator [13] corrections or those satisfying generalized universality [14]. This concept of universality can be generalized to certain vertex corrections with the non-standard weak boson interactions [15]. It is also often useful in theories with intrinsic vertex and box corrections, such as the SUSY-SM, since the propagator corrections are generally larger than the vertex/box ones: propagator corrections can be significant either because of a large multiplicity of contributing particles or by a presence of a relatively light new particle, whereas the vertex and box corrections depend on a specific combination of new particles that match the quantum number of the process and are suppressed if one of them is heavy.

Our framework adopts this distinction between new physics contributions to the gauge boson propagators and the rest, where we allow the most general contributions in the former whereas we consider only the SM contributions to the latter (vertex and box corrections). The new physics degree of freedom is then expressed in terms of four charge form factors, each associated with the four types of the electroweak gauge boson propagators :

$$\bar{e}^2(q^2) = \hat{e}^2[1 - \text{Re} \bar{\Pi}_{T,\gamma}^{\gamma\gamma}(q^2)] \quad \text{for the } \gamma\gamma \text{ propagator,} \quad (1a)$$

$$\bar{s}^2(q^2) = \hat{s}^2[1 + \frac{\hat{c}}{\hat{s}} \text{Re} \bar{\Pi}_{T,\gamma}^{\gamma Z}(q^2)] \quad \text{for the } \gamma Z \text{ propagator,} \quad (1b)$$

$$\bar{g}_Z^2(q^2) = \hat{g}_Z^2[1 - \text{Re} \bar{\Pi}_{T,Z}^{ZZ}(q^2)] \quad \text{for the } ZZ \text{ propagator,} \quad (1c)$$

$$\bar{g}_W^2(q^2) = \hat{g}^2[1 - \text{Re} \bar{\Pi}_{T,W}^{WW}(q^2)] \quad \text{for the } WW \text{ propagator,} \quad (1d)$$

where the hatted couplings $\hat{e} = \hat{g}\hat{s} = \hat{g}_Z\hat{s}\hat{c}$ and the propagator functions are renormalized in the $\overline{\text{MS}}$ scheme. In addition to these four form factors we have the two weak boson masses m_W and m_Z as the parameters of the electroweak theory. Since the charge form factors are real continuous functions of q^2 , we have infinite degrees of free parameters when we use them to parametrize a theory. In practice, however, these charge form factors can be measured accurately enough only at specific q^2 ranges; all four of them at $q^2 = 0$ ($q^2 \ll m_Z^2$), and two of them, $\bar{s}^2(q^2)$ and $\bar{g}_Z^2(q^2)$, at $q^2 = m_Z^2$. Hence, we have just 8 parameters that are measured accurately to test a theory. Among these 8 parameters, three are known precisely; α , G_F and m_Z . Since the gauge boson properties are fixed at tree level by the three parameters in general models with the $\text{SU}(2) \times \text{U}(1)$ symmetry broken by a vacuum expectation value, we can use the remaining 5 parameters to test the theory at the quantum level: see Table 1. We therefore first determine the 5 parameters,

$\bar{s}^2(m_Z^2)$, $\bar{g}_Z^2(m_Z^2)$, $\bar{s}^2(0)$, $\bar{g}_Z^2(0)$, and $\bar{g}_W^2(0)$, from precision experiments, and then confront their values with various theoretical predictions.

Table 1

	accurately measured parameters	precisely known parameters	parameters to test a theory
	$\bar{e}^2(0)$ *	$\alpha = \bar{e}^2(0)/4\pi$	*
	$\bar{s}^2(0)$ $\bar{s}^2(m_Z^2)$		$\bar{s}^2(0)$ $\bar{g}_Z^2(m_Z^2)$
	m_Z $\bar{g}_Z^2(0)$ $\bar{g}_Z^2(m_Z^2)$	m_Z	$\bar{s}^2(0)$ $\bar{g}_Z^2(m_Z^2)$
	m_W $\bar{g}_W^2(0)$ *	$4\sqrt{2}G_F = \frac{\bar{g}_W^2(0)}{m_W^2} (1 + \bar{\delta}_G)$	$\bar{g}_W^2(0)$ *

When the new physics scale is significantly higher than the scale ($\lesssim m_Z^2$) of precision measurements, we can often neglect new physics contributions to the running of the charge form factors. Among our 5 parameters, the values of $\bar{s}^2(0)$ and $\bar{g}_Z^2(0)$ can then be determined from $\bar{s}^2(m_Z^2)$ and $\bar{g}_Z^2(m_Z^2)$, respectively, by the SM physics only. The effective number of the free parameters is then 3, which corresponds precisely to that of S , T , U [11], ϵ_1 , ϵ_2 , ϵ_3 [13], or other related triplets of parameters in refs. [12]. When the scale of new physics that couples to gauge boson propagators is near the weak boson masses, we can identify its signal as an anomalous running of the charge form factors. This point has been emphasized in refs. [16] in connection with possible existence of the light SUSY particles. The triplet parametrizations are then no longer sufficient to account for new physics degrees of freedom, and we should regard all 5 parameters in the Table 1 as free parameters. Several alternative approaches to the same problem have been proposed in refs. [16–18].

Even if new physics scale is large, there can appear an anomalous running of the charge form factors. In fact, when new physics is parametrized in terms of the 4 gauge invariant dimension-six operators [14], O_{DB} , O_{DW} , O_{BW} , and $O_{\phi,1}$ in the notation of [15], then the new physics associated with the operators O_{DB} and O_{DW} contribute to the running of all the charge form factors [15]. Hence the operator formalism of ref. [14] is not equivalent to the approaches with 3 oblique parameters [11–13], but is comfortably accommodated within our framework.

A clear advantage of this approach is that we can test the electroweak theory at qualitatively different levels. If we find an inability to fit all the data at a given q^2 with common form factor values, we should either look for new physics that affect the relevant vertex/box corrections significantly or else we should introduce new tree level interactions such as those induced by an exchange of a new heavy boson. If the ‘universality’ in terms of the above four charge form factors holds, but their q^2 -dependence does not agree with the expectations of the standard model, we may anticipate a new physics scale very near to the present experimental limit. Hence new physics contributions that decouple at low energies ($\sim q^2/M^2$) can be identified as an anomalous running of the charge form factors. Finally, if even the running of the form factors is found to be consistent with the SM, then our approach reduces to the standard three parameter analyses [11–13]. Deviation from the SM is still possible since the SM has only two free parameters, m_t and m_H . Here we have sensitivity to those new physics contributions that do not decouple at low energies.

In the minimal SM, all the quantum corrections are determined by just two parameters, m_t and m_H , and hence all the charge form factors are determined by their values. We show in Fig. 1 the four charge form factors in the SM. The trajectories are fixed such that they give correct values for the 3 precisely known parameters, α , G_F , and m_Z . We show 12 trajectories each for six combination of the mass values, $m_t = 100, 150, 200$ GeV, $m_H = 100, 1000$ GeV, and for space-like ($q^2 < 0$) and time-like ($q^2 > 0$) momenta. The electric charge form factor $\bar{\alpha}(q^2) = \bar{e}^2(q^2)/4\pi$ does not depend on m_H . The threshold singularities are clearly seen in the time-like trajectories. Light hadron threshold effects do not show up since we adopt the dispersion integral fit of the hadronic contributions to the vacuum polarizations in the space-like region [19, 20] also for their contribution in the time-like region.

Fig. 1: Charge form factors in the minimal SM.

The 5 parameters that we determine from precision experiments are also shown as 'data' points in the figures. It is clear that these 'data' are perfectly consistent with the predictions of the minimal SM, for a certain (m_t, m_H) range, and that no indication of new physics is found. It should be noted here that there is no good measurement of the charge form factors $\bar{e}^2(q^2)$ and $\bar{g}_W^2(q^2)$ except at low energies $q^2 \sim 0$. We may expect TRISTAN and HERA to measure them and the W widths measure $\bar{g}_W^2(m_W^2)$, but it is challenging to achieve an accuracy comparable to those achieved in the low energy neutral current experiments ($\bar{s}^2(0)$ and $\bar{g}_Z^2(0)$).

We now explain some technical details of our framework. Those who are interested only in the results of our analysis may skip to section 3.

The gauge boson two-point functions that appear in eq.(1) are defined as follows:

$$\overline{\Pi}_{T,V}^{AB}(q^2) = \frac{\overline{\Pi}_T^{AB}(q^2) - \overline{\Pi}_T^{AB}(m_V^2)}{q^2 - m_V^2}, \quad (2)$$

where m_V is the pole mass of the gauge boson V ($m_\gamma = 0$) and the subscript T stands for the transverse part of the vacuum polarization tensor $\Pi_{\mu\nu}(q)$. The effective charge form factors of eq.(1) naturally appear in the S -matrix elements of the gauge boson exchange processes with external light fermions, which can be shown schematically as follows. The Dyson summation of the one particle irreducible propagator factors $\Pi_T^{VV}(q^2)$ gives the full VV propagator at the one-loop level

$$G_T^{VV}(q^2) = \frac{1}{q^2 - \hat{m}_V^2 + \Pi_T^{VV}(q^2)}, \quad (3)$$

where \hat{m}_V is the bare mass of the vector boson V . The physical mass and the width is then obtained as the pole position of the above full propagator:

$$m_V^2 - im_V\Gamma_V = \hat{m}_V^2 - \Pi_T^{VV}(m_V^2 - im_V\Gamma_V), \quad (4)$$

which can be solved perturbatively. Consistent perturbative expansion of the full propagator is then obtained as

$$G_T^{VV}(q^2) = \frac{1}{q^2 - m_V^2 + im_V\Gamma_V} \{1 - \Pi_{T,V}^{VV}(q^2)\}, \quad (5)$$

and the S -matrix elements contain the effective charge factors of eq.(1).

The propagators are calculated in the 'tHooft-Feynman gauge and the so-called pinch term [8, 21, 22] of the vertex functions due to diagrams with the weak boson self-couplings are included in the overlined functions $\overline{\Pi}_T^{AB}(q^2)$:

$$\overline{\Pi}_T^{\gamma\gamma}(q^2) = \Pi_T^{\gamma\gamma}(q^2) - \frac{\hat{e}^2}{4\pi^2} q^2 B_0(q^2; m_W, m_W), \quad (6a)$$

$$\overline{\Pi}_T^{\gamma Z}(q^2) = \Pi_T^{\gamma Z}(q^2) - \frac{\hat{e}\hat{g}_Z\hat{c}^2}{8\pi^2} (2q^2 - m_Z^2) B_0(q^2; m_W, m_W), \quad (6b)$$

$$\overline{\Pi}_T^{ZZ}(q^2) = \Pi_T^{ZZ}(q^2) - \frac{\hat{g}_Z^2\hat{c}^4}{4\pi^2} (q^2 - m_Z^2) B_0(q^2; m_W, m_W), \quad (6c)$$

$$\overline{\Pi}_T^{WW}(q^2) = \Pi_T^{WW}(q^2) - \frac{\hat{g}^2}{4\pi^2} (q^2 - m_W^2) [\hat{c}^2 B_0(q^2; m_W, m_Z) + \hat{s}^2 B_0(q^2; m_W, m_\gamma)]. \quad (6d)$$

Here B_0 is a Passarino-Veltman propagator function [23] in the notation of ref. [24]. There are two advantages in absorbing the above q^2 dependent propagator-like parts of the vertex functions into the effective charges [8], as compared to the standard ones [10] that absorb the relevant vertex term at zero momentum transfer. One is that the remaining vertex parts no more give rise to large logarithms of the type $\ln(-q^2/m_W^2)$, and hence the effective charges are useful in making the improved Born approximation [8] even at

very high energies ($|q^2| \gg m_W^2$). The second is that the effective charges are now gauge invariant [8, 22], and hence we can discuss their properties independently of the other corrections of the same order which are process specific. Most importantly, we can obtain explicitly renormalization group invariant relations between the $\overline{\text{MS}}$ couplings and the effective charges

$$\frac{1}{\bar{e}^2(q^2)} = \frac{1}{\hat{e}^2(\mu)} [1 + \text{Re} \bar{\Pi}_{T,\gamma}^{\gamma}(q^2)], \quad (7a)$$

$$\bar{s}^2(q^2) = \hat{s}^2(\mu) + \frac{\bar{e}^2(q^2)}{\hat{e}(\mu)\hat{g}_Z(\mu)} \text{Re} \bar{\Pi}_{T,\gamma}^Z(q^2), \quad (7b)$$

within the 'tHooft-Feynman gauge of the electroweak theory. The trajectories of all the $\overline{\text{MS}}$ couplings ($\hat{e} = \hat{g}\hat{s} = \hat{g}_Z\hat{s}\hat{c}$) are completely fixed by the above two equations, which can be used to study quantitatively the heavy particle threshold corrections in GUT theories [3, 25].

In our analysis we adopt the $\overline{\text{MS}}$ couplings as the expansion parameters of the perturbation series, since we find them most convenient when studying consequences of various theoretical models beyond the SM. Their usefulness in the SM analysis has been emphasized in ref. [26]. However, it is not convenient to use the $\overline{\text{MS}}$ couplings at a specific unit-of-mass (μ) scale, such as $\mu = m_Z$, when dealing with a theory with particles much heavier than the weak bosons because of the appearance of large logarithms of their masses. We hence take the following renormalization conditions

$$\hat{e}^2 = \bar{e}^2(m_Z^2), \quad \hat{s}^2 = \bar{s}^2(m_Z^2), \quad (8)$$

consistently for all processes that we study. The above conditions renormalize all the logarithms of large masses with the help of the renormalization group identities (7). We note here that the running of $\bar{e}^2(q^2)$ and $\bar{s}^2(q^2)$ at low energies as observed in Fig. 1 is due to the QED interactions [27], and hence the ratio $\bar{e}^2(q^2)/\bar{s}^2(q^2)$ is not an appropriate expansion parameter of the weak corrections even at $|q^2| \ll m_Z^2$. We further note that, apart from details concerning the higher order terms, our effective charges $\bar{e}^2(q^2)$ and $\bar{s}^2(q^2)$ are the same as the real parts of the corresponding star-scheme [8] charges, $e_*^2(q^2)$ and $s_*^2(q^2)$, respectively.

3. PRECISION EXPERIMENTS

All the precision experiments that are sensitive to electroweak physics at the one loop level have so far been confined to those processes with external light quarks and leptons, where their masses can safely be neglected as compared to the weak boson masses. They are the Z boson properties as measured at LEP and SLC, the neutral current (NC) processes at low energies ($\ll m_Z$), the charged current (CC) processes at low energies and the W mass measurements at the $p\bar{p}$ colliders. The relevant observables in these processes are expressed in terms of the S -matrix elements of four external light fermions which form a scalar product of two chirality conserving currents. All the information on the electroweak physics can be learned by studying the scalar amplitude multiplying these current-current products.

For example, we parameterize the S -matrix element of the NC process $ij \rightarrow ij$ (or any one of its crossed channels) as

$$T_{ij} = M_{ij} J_i \cdot J_j, \quad (9)$$

where J_i^μ and J_j^μ denote the bare currents without the coupling factor: $J_i^\mu = \bar{\psi}_f \gamma^\mu P_\alpha \psi_f$ for $i = f_\alpha$, where $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$ are the chiral projectors. All the one-loop corrections appear in the scalar amplitude M_{ij} which depends on the invariant momentum transfers s and t .

In the neutral current amplitudes, the photonic corrections attached only to the external fermion lines are gauge invariant in themselves. Therefore we can obtain finite and gauge invariant amplitudes by excluding all the external photonic corrections. By using the charge form factors of eq.(1), we find e.g. for the process $i\bar{i} \rightarrow j\bar{j}$

$$\begin{aligned} M_{ij}^{NC} = & \frac{Q_i Q_j}{s} [\bar{e}^2(s) + \hat{e}^2(\Gamma_1^i + \Gamma_1^j)(s) - i\hat{e}^2 Im \bar{\Pi}_{T,\gamma}^{\gamma\gamma}(s)] + \hat{e}^2 [(Q_i I_{3j}) \frac{\bar{\Gamma}_2^j(s)}{s} + (I_{3i} Q_j) \frac{\bar{\Gamma}_2^i(s)}{s}] \\ & + \frac{1}{s - m_Z^2 + i s \frac{\Gamma_Z}{m_Z}} \{ (I_{3i} - Q_i \hat{s}^2)(I_{3j} - Q_j \hat{s}^2) [\bar{g}_Z^2(s) + \hat{g}_Z^2(\Gamma_1^i + \Gamma_1^j)(s) - i\hat{g}_Z^2 \Delta_Z(s)] \\ & + (I_{3i} - Q_i \hat{s}^2) \hat{g}_Z^2 [I_{3j} (\hat{c}^2 \bar{\Gamma}_2^j + \Gamma_3^j)(s) + \Gamma_4^j(s) - Q_j (\bar{s}^2(s) - \hat{s}^2 + i\hat{s} \hat{c} \Delta_{\gamma Z}(s))] \\ & + (I_{3j} - Q_j \hat{s}^2) \hat{g}_Z^2 [I_{3i} (\hat{c}^2 \bar{\Gamma}_2^i + \Gamma_3^i)(s) + \Gamma_4^i(s) - Q_i (\bar{s}^2(s) - \hat{s}^2 + i\hat{s} \hat{c} \Delta_{\gamma Z}(s))] \} \\ & + B_{ij}^{NC}(s, t). \end{aligned} \quad (10)$$

The vertex functions $\Gamma_n^{f_\alpha}(s)$ and the box functions $B_{f_\alpha f'_\beta}(s, t)$ are process specific. We first note that the residues of the γ and Z poles are separately physical observables (μ -independent and gauge invariant). At $q^2 = 0$, we find

$$\Gamma_1^{f_\alpha}(0) = \bar{\Gamma}_2^{f_\alpha}(0) = 0 \quad (11)$$

for all f_α , which are ensured by the Abelian and non-Abelian parts of the Ward identities, respectively. The universal residue of the photon pole gives the square of the unit electric charge $\bar{e}^2(0) = 4\pi\alpha$.

A few technical comments are in order. In eq.(10), the matrix elements are linear in the one-loop functions where the renormalization group improvement is achieved by (7) and the condition (8). The use of the running Z width above necessarily modifies the mass renormalization conditions (4) [28], and we adopt the convention of ref. [29] for m_Z . The associated small changes in the propagator correction factors (Δ_Z and $\Delta_{\gamma Z}$) are not given explicitly above for brevity. The overlines on the vertex functions $\bar{\Gamma}_2^{f_\alpha}(s)$ indicates the removal of the pinch term [8, 22]. The vertex functions $\Gamma_3^{f_\alpha}(s)$ are proportional to the square of the fermion mass inside the loop, and are non-vanishing only for $f_\alpha = b_L$ in the SM. The functions $\Gamma_4^{f_\alpha}(s)$ are vanishing for all f_α in the SM, though they appear in extended models such as the minimal SUSY-SM. The box functions $B_{ij}(s, t)$ are needed only at low energy NC processes at $s = t = 0$. It is worth noting here that the box contributions to the helicity amplitudes can be expressed in the above simple current product form only when the external fermion masses can be neglected. All the vertex and box functions are known precisely in the SM. If we assume no new physics contributions to

these process specific (f_α -dependent) corrections, we can determine the three form factors $\bar{e}^2(q^2)$, $\bar{g}_Z^2(q^2)$ and $\bar{s}^2(q^2)$ from the precision experiments independent of further model assumptions.

For the charged current (CC) process $ij \rightarrow i'j'$, we find similarly

$$M_{ij}^{CC} = \frac{1}{t - m_W^2} \{ \bar{g}_W^2(t) + \hat{g}^2 [\Gamma_1^{ii'} + \Gamma_1^{jj'} + \bar{\Gamma}_2^{ii'} + \bar{\Gamma}_2^{jj'}](t) \} + B_{ij}^{CC}(s, t), \quad (12)$$

off the W pole, with an appropriate CKM factor $V_{ii'}V_{jj'}^*$. Precise values of the CC matrix elements are needed only at low energies, and we find for the muon decay constant

$$G_F = \frac{\bar{g}_W^2(0) + \hat{g}^2 \bar{\delta}_G}{4\sqrt{2}m_W^2}. \quad (13)$$

Here the factor $\bar{\delta}_G$ denotes the sum of the vertex and the box contributions, whose value is precisely known ($\bar{\delta}_G = 0.0055$) in the SM. Eq.(13) gives the physical W mass in terms of G_F once the $\bar{\delta}_G$ value is known for a given model. The overline here again indicates the removal of the pinch terms and that its numerical value is significantly (about 25%) smaller than the standard factor [30].

In the following, we assume that there are no new physics contributions to the vertex and box corrections, except that we allow the $Zb_L b_L$ vertex to take an arbitrary value, and determine the form factors (1) from the three sectors of the electroweak precision experiments.

3.1 Z boson parameters

The most recent results from experiments at LEP and SLC on the Z boson parameters have been reported in refs. [31,32]. The Z line-shape parameters are determined at LEP as [32]

$$\begin{aligned} m_Z(\text{GeV}) &= 91.187 \pm 0.007 \\ \Gamma_Z(\text{GeV}) &= 2.489 \pm 0.007 \\ \sigma_h^0(\text{nb}) &= 41.56 \pm 0.14 \\ R_\ell = \sigma_h^0/\sigma_\ell^0 &= 20.763 \pm 0.049 \\ A_{\text{FB}}^{0,\ell} &= 0.0158 \pm 0.0018 \end{aligned} \quad \rho_{\text{corr}} = \begin{pmatrix} 1 & -0.157 & 0.017 & 0.012 & 0.075 \\ & 1 & -0.070 & 0.003 & 0.006 \\ & & 1 & 0.137 & 0.003 \\ & & & 1 & 0.008 \\ & & & & 1 \end{pmatrix}. \quad (14)$$

The other electroweak data that we used in our fit are as follows [31,32]:

$$P_\tau = -0.139 \pm 0.014, \quad (15a)$$

$$A_{\text{LR}} = 0.10 \pm 0.044 \quad (\text{SLD [33]}), \quad (15b)$$

$$A_{\text{FB}}^{0,b} = 0.099 \pm 0.006, \quad (15c)$$

$$A_{\text{FB}}^{0,c} = 0.075 \pm 0.015, \quad (15d)$$

$$R_b = \sigma_b^0/\sigma_h^0 = 0.2203 \pm 0.0027 \quad (\text{LEP} + \text{SLD}). \quad (15e)$$

Significant improvements over the last year have been achieved for many of the above measurements.

In order to determine the universal charge form factors $\bar{s}^2(m_Z^2)$ and $\bar{g}_Z^2(m_Z^2)$ from these data, we should estimate the SM corrections to the vertex diagrams, QCD higher

order effects, and external fermion mass effects. We assume that only three neutrinos ($N_\nu = 3$) contribute to the invisible width of Z , and take the standard perturbative QCD corrections for the vector [34] and axial-vector [35] Z couplings, the quark mass effects [36] and the forward-backward asymmetries [37].

The fit results are then found to depend on the assumed α_s value and on m_t which affects the Zb_Lb_L vertex function [38]. In the absence of an accurate quantitative measurement of the QCD coupling constant and for the convenience of the GUT studies, we choose α_s as a free parameter of our fit, and present the α_s dependences of the minimal χ^2 values. One can either add independent data on α_s or study quantitative consequences of a particular GUT model that predicts α_s .

As for the strong m_t dependence of the Zb_Lb_L vertex, we find it convenient to introduce one extra form factor

$$\bar{\delta}_b(s) = \Gamma_1^{b_L}(s) + \hat{c}^2 \bar{\Gamma}_2^{b_L}(s) + \Gamma_3^{b_L}(s) \quad (16)$$

in our fit. A similar strategy has been proposed in ref. [17]. An advantage is that the parameter $\bar{\delta}_b$ allows us to determine the quantitative significance of the Zb_Lb_L vertex correction [39], independent of the specific SM mechanism. Furthermore, it allows us to separate the data analysis stage from the evaluation of $\bar{\delta}_b$ in a specific model, that includes $O(\alpha_s m_t^2)$ [40] and $O(m_t^4)$ [41, 42] two-loop corrections.

The overall fit to all the Z parameters listed above in terms of the three parameters $\bar{s}^2(m_Z^2)$, $\bar{g}_Z^2(m_Z^2)$ and $\bar{\delta}_b(m_Z^2)$ for a given value of α_s gives

$$\begin{aligned} \bar{g}_Z^2(m_Z^2) &= 0.5546 - 0.031(\alpha_s - 0.12) \pm 0.0017 \\ \bar{s}^2(m_Z^2) &= 0.2313 + 0.008(\alpha_s - 0.12) \pm 0.0007 \\ \bar{\delta}_b(m_Z^2) &= -0.0061 - 0.430(\alpha_s - 0.12) \pm 0.0035 \end{aligned} \quad \rho_{\text{corr}} = \begin{pmatrix} 1 & 0.14 & -0.36 \\ & 1 & 0.20 \\ & & 1 \end{pmatrix} \quad (17a)$$

$$\chi_{\text{min}}^2 = 1.60 + ((\alpha_s - 0.103)/0.0127)^2, \quad (17b)$$

where the errors and the correlations are almost independent of α_s .

The above results are shown in Fig. 2, along with the SM predictions with all known corrections to the ρ -parameter [43] in the $O(m_t^4)$ level [41, 44–46] and the $O(\alpha_s)$ two-loop corrections [47] in perturbative QCD, but without non-perturbative $t\bar{t}$ threshold effects [48]. The SM prediction to $\bar{s}^2(m_Z^2)$ is also sensitive to the hadronic vacuum polarization correction, for which we take [20] $(\Delta_{\alpha}^{\frac{1}{2}})_{\text{hadrons}} = -0.0283/\alpha = -3.88$. Its error $\delta_{\alpha} = \pm 0.0007/\alpha = \pm 0.10$ leads to a shift in the SM predictions for $\bar{s}^2(m_Z^2)$ by ± 0.00026 .

We show in Fig. 2 1- σ contours of the fit for three representative α_s values. It is clearly seen that the γZ -mixing parameter $\bar{s}^2(m_Z^2)$ is measured rather insensitively to α_s , while the Z coupling strength $\bar{g}_Z^2(m_Z^2)$ is negatively correlated with the assumed α_s value, reflecting its sensitivity to the total Z width. This anti-correlation leads to a preference of larger m_t in the SM for smaller α_s . The relative insensitivity of the parameter $\bar{s}^2(m_Z^2)$ to α_s can easily be understood since it can be measured from the asymmetry parameters that are either completely or almost insensitive to the QCD corrections. We list below its value determined from each asymmetry measurement:

$$\bar{s}^2(m_Z^2) = 0.2309 \pm 0.0010 \quad (\text{from } A_{\text{FB}}^{0,\ell}), \quad (18a)$$

$$\bar{s}^2(m_Z^2) = 0.2316 \pm 0.0018 \quad (\text{from } P_{\tau}), \quad (18b)$$

$$\bar{s}^2(m_Z^2) = 0.2365 \pm 0.0055 \quad (\text{from } A_{\text{LR}}), \quad (18c)$$

$$\bar{s}^2(m_Z^2) = 0.2313 + 0.004(\alpha_s - 0.12) \pm 0.0011 \quad (\text{from } A_{\text{FB}}^{0,b}), \quad (18d)$$

where the first three lepton asymmetries are almost completely insensitive to $\bar{g}_Z^2(m_Z^2)$ or α_s , and the b -quark forward-backward asymmetry is also insensitive to $\bar{g}_Z^2(m_Z^2)$ or $\bar{\delta}_b(m_Z^2)$ while mildly sensitive to α_s due to perturbative QCD correction [37]. From the above data alone, we find

$$\bar{s}^2(m_Z^2) = 0.2312 \pm 0.0009 \quad (\text{from } A_{\text{FB}}^{0,\ell}, P_\tau, A_{\text{LR}}), \quad (19a)$$

$$\bar{s}^2(m_Z^2) = 0.2312 + 0.002(\alpha_s - 0.12) \pm 0.0007 \quad (\text{from } A_{\text{FB}}^{0,\ell}, P_\tau, A_{\text{LR}}, A_{\text{FB}}^{0,b}). \quad (19b)$$

We may expect a significantly improved measurement of A_{LR} from SLD in the near future [31]. τ polarization measurement may still be improved [49]. These asymmetry measurements are particularly important for GUT studies, since the parameter $\bar{s}^2(m_Z^2)$ is directly related to the unifying coupling $\hat{s}^2(\mu)$ via eq.(7).

Fig. 2: Three parameter fits to the Z boson parameters for three $\alpha_s(m_Z)$ values. Also shown are and the SM predictions for $(\Delta_{\alpha}^{\frac{1}{\alpha}})_{\text{hadrons}} = -3.88$ ($\delta_{\alpha} = 0$) [20].

Before leaving the Z parameters, we would like to give two comments on the measurements of the $Zb_L b_L$ vertex and α_s , which are strongly correlated. As is clearly seen from Fig. 2, the fit to the parameter $\bar{\delta}_b$ depends strongly on α_s , reflecting its sensitivity to R_l and Γ_Z , in addition to R_b that is rather insensitive to α_s . Because of this sensitivity to α_s , it is not meaningful to quote a bound on $\bar{\delta}_b$, or on m_t in the SM, without studying carefully its α_s dependence. It is worth emphasizing here that there is no evidence of the $Zb_L b_L$ vertex for $\alpha_s \gtrsim 0.13$, as the corresponding parameter for d_L or s_L is about -0.003 .

For $\alpha_s \gtrsim 0.12$, we can obtain rather stringent upper bound on m_t [17, 39] that one can read off from Fig. 2, mainly because there is no good evidence for the $Zb_L b_L$ vertex effect. This point has also been emphasized by the LEP electroweak working group [32]. Furthermore, this strong correlation makes the fitted α_s value depend strongly on the assumed $\bar{\delta}_b$ value. If we allow $\bar{\delta}_b$ and α_s to be fitted freely by the data, then the result (17) gives $\bar{\delta}_b(m_Z^2) = 0.0015 \pm 0.0071$ and $\alpha_s(m_Z^2) = 0.103 \pm 0.013$, with $\rho_{\text{corr}} = -0.85$. It is therefore necessary to assume the SM contributions to $\bar{\delta}_b(m_Z^2)$, and to a lesser extent those to $\bar{g}_Z^2(m_Z^2)$, in order to measure α_s from the electroweak Z -parameters. The result of such an analysis is given in section 4.3 where we study consequences of the minimal SM.

3.2 Low energy neutral current experiments

We consider in our analysis four types of low energy neutral current experiments. They are the neutrino-nuclei scattering ($\nu_\mu - q$), the neutrino-electron scattering ($\nu_\mu - e$), atomic parity violation (APV), and the polarized electron-deuteron scattering experiments (eD). All of them measure the universal form factors $\bar{s}^2(0)$ and $\bar{g}_Z^2(0)$. Effects due to small but finite momentum transfer in these processes are corrected for by assuming that the running of these form factors are determined by the SM particles only (see Fig. 1), which is an excellent approximation at low energies. Vertex and box corrections are performed by assuming that they are dominated by the SM contributions. For each sector, we first give a model-independent parametrization of the data, and then give our fit in the $(\bar{s}^2(0), \bar{g}_Z^2(0))$ plane.

For the $\nu_\mu - q$ data, we used the results of the analysis of ref. [50]. The fitted parameters ($g_L^2, g_R^2, \delta_L^2, \delta_R^2$) are, however, dependent on the assumed value of the charmed quark mass (m_c) in the slow-rescaling formula for the charged current cross sections. By using the constraint on m_c from the charged current experiments, $m_c = 1.54 \pm 0.33$ GeV [50], we can properly take into account the m_c dependence of the fit. We thus find a new model-independent parametrization of the $\nu_\mu - q$ data:

$$\begin{aligned} g_L^2 &= 0.2980 \pm 0.0044 \\ g_R^2 &= 0.0307 \pm 0.0047 \\ \delta_L^2 &= -0.0589 \pm 0.0237 \\ \delta_R^2 &= 0.0206 \pm 0.0160 \end{aligned} \quad \rho_{\text{corr}} = \begin{pmatrix} 1 & -0.559 & -0.163 & 0.162 \\ & 1 & 0.156 & -0.037 \\ & & 1 & -0.447 \\ & & & 1 \end{pmatrix}. \quad (20)$$

The standard model fit is then performed by expressing the above parameters in terms of the ratio of the squares of the NC and CC S -matrix elements of eqs.(10,12) evaluated at $< -t >_{NC} = < -t >_{CC} = 20 \text{ GeV}^2$. We reproduced the well-known results of ref. [51]. The corrections due to the running of $\bar{s}^2(t)$, the neutrino 'charge radius' factor [52] $\bar{\Gamma}_2^\mu(t)$ of eq.(10), and the WW box are found to be significant. After further correcting for the QED radiation effects in the CC cross section [53], we find

$$\left. \begin{aligned} \bar{g}_Z^2(0) &= 0.5486 \pm 0.0080 \\ \bar{s}^2(0) &= 0.2398 \pm 0.0143 \end{aligned} \right\} \quad \rho_{\text{corr}} = 0.92, \quad (21a)$$

$$\chi_{\text{min}}^2 = 0.86. \quad (21b)$$

The strong positive correlation is a consequence of the smallness of the error of $g_L^2 + g_R^2$ in (20) that measures the total neutral current cross section off isoscalar targets. The above fit is given in Fig. 3 as a 1- σ counter.

For the $\nu_\mu e$ data, we used the results of CHARM, BNL E374 and CHARM-II [54], which are summarized by R. Beyer [54] as

$$\left. \begin{aligned} (\rho)_{\text{eff}}^{\nu_\mu e} &= 1.007 \pm 0.028 \\ (\sin^2 \theta_W)_{\text{eff}}^{\nu_\mu e} &= 0.233 \pm 0.008 \end{aligned} \right\} \quad \rho_{\text{corr}} = 0.09. \quad (22)$$

These effective parameters are obtained from the data by assuming the tree-level formula for the $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering cross sections. We can hence obtain the electroweak parameters by evaluating the full matrix elements at an average momentum transfer of these experiments, $< -t > \sim m_\mu^2$. We reproduce the known results of ref. [55], and find that the only significant correction comes from the neutrino 'charge radius' factor and the WW box contributions. We find

$$\left. \begin{aligned} \bar{g}_Z^2(0) &= 0.5459 \pm 0.0153 \\ \bar{s}^2(0) &= 0.2416 \pm 0.0080 \end{aligned} \right\} \quad \rho_{\text{corr}} = 0.09 \quad (23)$$

with $\chi_{\text{min}}^2 = 0$, since we take the fit (22) as the model independent parametrization of the $\nu_\mu - e$ data [54]. The result is also shown in Fig. 3.

As for the APV experiments, we used the result of the analysis [56] on the parity violating transitions in the cesium atom (A,Z)=(135,55);

$$Q_W(135, 55) = -71.04 \pm 1.81 \quad (24)$$

where we sum the experimental and theoretical errors by quadrature. Our simple formula (10) reproduces the u - and d -quark contributions of ref. [57], but not the photonic correction to the axial vector Zee vertex nor the $Z\gamma$ box corrections that are sensitive to the nucleon structure. We adopt the results of ref. [57] for these corrections, and find

$$\bar{s}^2(0) = -0.6130 \cdot \bar{g}_Z^2(0) + 0.5661 \pm 0.0083 \quad (25)$$

The result is shown in Fig. 3.

Finally, for the SLAC eD polarization asymmetry experiment [58], we make a model-independent fit to the original data by using the two parameters, $2C_{1u} - C_{1d}$ and $2C_{2u} - C_{2d}$ of ref. [59], by taking into account uncertainties due to the sea-quark contributions and finite $R = \sigma_L/\sigma_T$ [60], and those due to higher twist contributions [61,62]. The former uncertainties are found to be very small, confirming the results of ref. [60], while the latter are found to be model dependent [63]. We adopt the estimates [62] based on the MIT-Bag model, which find rather small corrections, as in the neutrino scattering off isoscalar targets [64]. Further study on the higher twist effects may be needed to achieve precision measurements of the electroweak parameters in these reactions. After allowing for uncertainties in the Bag model parameters of ref. [62], we find

$$\left. \begin{aligned} 2C_{1u} - C_{1d} &= -0.94 \pm 0.26 \\ 2C_{2u} - C_{2d} &= 0.66 \pm 1.23 \end{aligned} \right\} \quad \rho_{\text{corr}} = -0.975 \quad (26)$$

with $\chi_{\text{min}}^2 = 9.95$ for 11 data points. Because of the strong correlation, only a linear combination of the two coupling factors is measured well. The electroweak corrections in the SM are found in ref. [65]. Our formula (10) leads to all relevant correction factors

except for the external photonic corrections. We use the explicit form of ref. [57] for these correction factors, and checked the insensitivity of our fit to the uncertainty in the $Z\gamma$ box corrections. The QED coupling $\bar{e}^2(t)$ and the vertex functions $\Gamma_1(t)$ and $\bar{\Gamma}_2(t)$ in our amplitudes (10) are evaluated at $\langle -t \rangle = 1.5\text{GeV}^2$. We find

$$\bar{s}^2(0) = 0.3264 \cdot \bar{g}_Z^2(0) + 0.0471 \pm 0.0094 \quad (27a)$$

$$\chi_{\min}^2 = -1.77 \cdot \bar{g}_Z^2(0) + 1.43, \quad (27b)$$

where χ_{\min}^2 is obtained by taking our model-independent fit (26) as our input 'data'.

The results of our two parameter fit to all the neutral current data are summarized in Fig. 3 by 1- σ allowed regions in the $(\bar{s}^2(0), \bar{g}_Z^2(0))$ plane. They are consistent with each other and, after combining the above four sectors, we find

$$\left. \begin{aligned} \bar{g}_Z^2(0) &= 0.5459 \pm 0.0035 \\ \bar{s}^2(0) &= 0.2350 \pm 0.0045 \end{aligned} \right\} \quad \rho_{\text{corr}} = 0.53, \quad (28a)$$

$$\chi_{\min}^2 = 2.82. \quad (28b)$$

The fit is excellent as the effective degrees of freedom of the fit is $8 - 2 = 6$. The combined fit above is shown by the thick 1- σ contour in Fig. 3.

Fig. 3: Two parameter fit to L.E.N.C. data.

Fig. 4: Z parameter fit plus L.E.N.C. fit.

3.3 Charged current experiments

The W mass data have been updated this summer by the CDF and D0 collaborations [66]. We obtain

$$m_W = 80.25 \pm 0.24\text{GeV} \quad (29)$$

by combining the two most recent measurements [66] after adding all the quoted errors by quadrature.

The electroweak parameter $\bar{g}_W^2(0)$ is then obtained from the μ life-time via the identity (13). By using the SM estimate $\bar{\delta}_G = 0.0055$ and the perturbative approximation $\hat{g}^2 = \bar{g}_W^2(0)$, we find

$$\bar{g}_W^2(0) = 0.4226 \pm 0.0025. \quad (30)$$

No other experiment in the charged current sector is accurate enough to add useful information in our electroweak analysis. Precision measurements of the W width [67] and its leptonic branching fraction may determine $\bar{g}_W^2(m_W^2)$ in the future.

4. SYSTEMATIC ANALYSIS

All the electroweak precision data have now been represented by the charge form factor values of eqs.(17,28,30). We find that all results are consistent with the assumptions of the $SU(2)_L \times U(1)_Y$ universality and the SM dominance of the vertex and box corrections. In the following, we perform the fit to the data in three steps by systematically strengthening the model assumptions.

4.1 Testing the running of the charge form factors

Only two of the four form factors, $\bar{s}^2(q^2)$ and $\bar{g}_Z^2(q^2)$, have been measured sufficiently accurately at two energy scales, $q^2 = 0$ and m_Z^2 . From eqs.(17,28), we find

$$\left. \begin{aligned} \bar{g}_Z^2(m_Z^2) - \bar{g}_Z^2(0) &= 0.0087 - 0.031(\alpha_s - 0.12) \pm 0.0039 \\ \bar{s}^2(m_Z^2) - \bar{s}^2(0) &= -0.0037 + 0.008(\alpha_s - 0.12) \pm 0.0046 \end{aligned} \right\} \quad \rho_{\text{corr}} = 0.48. \quad (31)$$

The SM predictions for these quantities are, respectively,

$$[\bar{g}_Z^2(m_Z^2) - \bar{g}_Z^2(0)]_{\text{SM}} = 0.00723 + 10^{-4} [2.5(\frac{m_t}{150} - 1) - 0.15 \ln \frac{m_H}{100} - 0.56(\frac{100}{m_H})^2], \quad (32a)$$

$$[\bar{s}^2(m_Z^2) - \bar{s}^2(0)]_{\text{SM}} = -0.00838 - 10^{-4} [3.1(\frac{m_t}{150} - 1) - 0.33 \ln \frac{m_H}{100}], \quad (32b)$$

where m_t and m_H are measured in GeV units. Both results are consistent at the 1- σ level with the assumption that the running of these form factors is governed by the SM particles only. Since the running of the form factors is affected only by particles of mass in the vicinity of m_Z , we conclude that there is no indication of new particles of mass $\lesssim m_Z$.

The errors in (31) are determined by those of the low energy experiments. Further improvements in the low energy precision experiments are needed to detect a signal of relatively light new particles.

4.2 Testing the three parameter universality

Once we assume further that the running of the charge form factors is governed by the SM particles only, then we can parametrize all the predictions of the general $SU(2)_L \times U(1)_Y$ model in terms of just three free parameters; see Table 1. This can easily be understood by noting that there are 6 parameters in the gauge boson sector of the model; the four charge form factors associated with the four gauge boson propagators and the two gauge boson masses m_W and m_Z . From these 6 parameters, 3 should be traded for the three fundamental parameters of the theory, the two gauge couplings and one vacuum expectation value. We choose for convenience the three most accurately measured quantities, α , G_F and m_Z , as the parameters which renormalize the sensitivity to physics at very high energies. The remaining 3 parameters can hence reveal one-loop physics at the weak scale. We first choose $\bar{s}^2(m_Z^2)$, $\bar{g}_Z^2(m_Z^2)$ and $\bar{g}_W^2(0)$ as the three parameters, and present the result of our global analysis. The result is then re-expressed in terms of another set of the parameters, S , T and U [11].

By using the SM running of the form factors (32), we can combine the Z parameter fit (17) and the low energy NC fit (28). This is schematically shown in Fig. 4, where the

combined low energy NC fit of Fig. 3 is reproduced in the $(\bar{s}^2(m_Z^2), \bar{g}_Z^2(m_Z^2))$ plane. The uncertainty in the running of the parameters within the SM is visualized by the thickness of the contour which spans the range $m_t = 100 - 200$ GeV, $m_H = 100 - 1000$ GeV in eq.(32). The low energy parameters are consistent with the Z parameters, which are also shown as the 'LEP+SLC' contour. All the neutral current data are now combined to give

$$\begin{aligned} \bar{g}_Z^2(m_Z^2) &= 0.5547 - 0.023(\alpha_s - 0.12) \pm 0.0015 \\ \bar{s}^2(m_Z^2) &= 0.2312 + 0.008(\alpha_s - 0.12) \pm 0.0007 \\ \bar{\delta}_b(m_Z^2) &= -0.0063 - 0.437(\alpha_s - 0.12) \pm 0.0034 \end{aligned} \quad \rho_{\text{corr}} = \begin{pmatrix} 1 & 0.16 & -0.31 \\ & 1 & 0.20 \\ & & 1 \end{pmatrix}, \quad (33a)$$

$$\chi_{\min}^2 = 5.40 + ((\alpha_s - 0.103)/0.0123)^2. \quad (33b)$$

The above fit is almost independent of (m_t, m_H) values assumed in the running of the charge form factors. The χ_{\min}^2 value of 7.3 for $\alpha_s = 0.12$ is excellent for the effective degrees of freedom of the fit, $18 - 3 = 15$.

There is one notable point at this stage which becomes apparent by comparing the global fit of Fig. 4 with the individual fit to low energy NC data in Fig. 3. Both the data on $\nu_\mu - q$ and $\nu_\mu - e$ experiments are perfectly consistent with the global fit, whereas the APV result and the eD asymmetry fit are just $1-\sigma$ away. Further studies of polarization asymmetries in the $e - q$ sector, as well as quantitative studies of the neutral current processes at TRISTAN energies might be potentially rewarding.

The global fit (33) for $(\bar{s}^2(m_Z^2), \bar{g}_Z^2(m_Z^2))$ from all the neutral current data and the fit (30) for $\bar{g}_W^2(0)$ from the charged current data summarize our knowledge on the electroweak parameters in our framework; see Table 1.

When the basic three parameters of the models with the $SU(2)_L \times U(1)_Y$ symmetry broken by just one vacuum expectation value are renormalized by the three well-known quantities α , G_F and m_Z , all the predictions of the theory are determined at the tree level. It is therefore convenient to introduce three parameters which are proportional to the finite quantum correction effects only. Among the various proposals in the literature [11–13], we find that the S , T , U parameters of Peskin and Takeuchi [11] is most convenient if they are extended to include the SM contributions as well. We *define* these parameters in terms of our two-point functions with the pinch terms [22], which are related to our charge form factors as follows:

$$S \equiv \frac{1}{\pi} \text{Re}[\bar{\Pi}_{T,\gamma}^{3Q}(m_Z^2) - \bar{\Pi}_{T,Z}^{33}(0)] = \frac{4\bar{s}^2(m_Z^2)\bar{c}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{16\pi}{\bar{g}_Z^2(0)}, \quad (34a)$$

$$\alpha T \equiv \frac{G_F}{2\sqrt{2}\pi^2} [\bar{\Pi}_T^{33}(0) - \bar{\Pi}_T^{11}(0)] = 1 + \bar{\delta}_G - \frac{4\sqrt{2}G_F m_Z^2}{\bar{g}_Z^2(0)}, \quad (34b)$$

$$S + U \equiv \frac{1}{\pi} \text{Re}[\bar{\Pi}_{T,\gamma}^{3Q}(m_Z^2) - \bar{\Pi}_{T,Z}^{11}(0)] = \frac{4\bar{s}^2(m_Z^2)}{\bar{\alpha}(m_Z^2)} - \frac{16\pi}{\bar{g}_W^2(0)}. \quad (34c)$$

These definitions allow us to express all the charge form factors and hence all experimental observables in terms of the three parameters S , T and U without separating the SM contributions to the gauge boson propagators. First, the form factor $\bar{g}_Z^2(0)$ is determined from T via eq.(34b). Second, the form factor $\bar{s}^2(m_Z^2)$ is determined from S via eq.(34a). And finally the form factor $\bar{g}_W^2(0)$ is determined from U via eq.(34c). The running of these form factors is determined by their defining equations (1) by properly performing

the renormalization group improvement via eq.(7). All the form factors are thus easily calculable for arbitrary models for fixed $(\alpha, G_F \text{ and } m_Z)$. In fact, the SM curves in Figs. 1 and 2 are obtained this way.

By assuming only the SM contribution to the muon decay vertex and box corrections in $\bar{\delta}_G$ and by assuming the SM running of the form factors, especially for $\bar{\alpha}(q^2)$, we can express all the charge form factors as a power series in the above three parameters. To first order, we find

$$\bar{g}_Z^2(0) = 0.5456 + 0.0040T, \quad (35a)$$

$$\bar{s}^2(m_Z^2) = 0.2334 + 0.0036S - 0.0024T - 0.0026\delta_\alpha, \quad (35b)$$

$$\bar{g}_W^2(0) = 0.4183 - 0.0031S + 0.0044T + 0.0035U - 0.0015\delta_\alpha, \quad (35c)$$

where we added the shifts due to the uncertainty in the estimated $1/\bar{\alpha}(m_Z^2)$ value, δ_α , which is as large as ± 0.10 [20]. It is clearly seen that $\bar{g}_Z^2(0)$ measures T , $\bar{s}^2(m_Z^2)$ measures a combination of S and T , whereas $\bar{g}_W^2(0)$ measures a combination of all three parameters.

We can now express the result of our global fit (33) and (30), in terms of the S , T , U parameters and $\bar{\delta}_b(m_Z^2)$. We find

$$\begin{aligned} S &= -0.29 - 1.6(\alpha_s - 0.12) + 0.73\delta_\alpha \pm 0.33 \\ T &= 0.46 - 5.8(\alpha_s - 0.12) - 0.04\delta_\alpha \pm 0.37 \\ U &= 0.39 + 5.8(\alpha_s - 0.12) + 0.25\delta_\alpha \pm 0.76 \\ \bar{\delta}_b &= -0.0063 - 0.44(\alpha_s - 0.12) \pm 0.0034 \end{aligned} \quad \rho_{\text{corr}} = \begin{pmatrix} 1 & -0.83 & -0.13 & -0.12 \\ & 1 & -0.29 & -0.31 \\ & & 1 & 0.14 \\ & & & 1 \end{pmatrix}, \quad (36a)$$

$$\chi_{\text{min}}^2 = 5.40 + ((\alpha_s - 0.103)/0.0124)^2 + (\delta_\alpha/0.1)^2. \quad (36b)$$

Only the correlation between the errors in S and T is significant. We show in Fig. 5 the above results on the (S, T) and (U, T) planes. The only radiative effect which is significantly non-vanishing is in the T parameter. Both the S and U parameters are consistent with zero at the 1- σ level. Note also that the S parameter is particularly sensitive to the hadronic uncertainty δ_α of $1/\bar{\alpha}(m_Z^2)$, whose magnitude can change by a quarter for $\delta_\alpha = \pm 0.10$ [20].

Fig. 5: Global fit to the (S, T, U) parameters as defined in eq.(34) for three α_s values and for (m_t, m_H) values in the range $m_t = 100 - 200$ GeV and $m_H = 100 - 1000$ GeV, for $(\Delta \frac{1}{\alpha})_{\text{hadron}} = -0.0283/\alpha = -3.88$ ($\delta_\alpha = 0$) [20]. The SM predictions are also given.

4.3 Testing the minimal standard model

In the minimal SM, the three parameters S , T , U as defined above and the Zb_Lb_L vertex form factor $\bar{\delta}_b(m_Z^2)$ are uniquely determined in the one-loop level by the two mass parameters m_t and m_H . We show in Fig. 6 the SM predictions for these parameters as functions of m_t for selected values of m_H , by including all the known two-loop corrections of $O(m_t^4)$ [41, 42, 46] and of $O(\alpha_s)$ [40, 47] at $\alpha_s(m_Z) = 0.12$. From Figs. 5 and 6, one can see that the parameters S and T show mild sensitivity to m_H , but the parameters U and $\bar{\delta}_b(m_Z^2)$ are almost independent of m_H .

Fig. 6: The SM predictions for the $(S, T, U, \bar{\delta}_b)$ parameters as defined in eqs.(34,16) as functions of m_t for selected m_H values. We set $\alpha_s = 0.12$ in the two-loop $O(\alpha_s)$ corrections for S, T, U [47] and $\bar{\delta}_b(m_Z^2)$ [40].

By inserting these SM (m_t, m_H) dependences into our global fits (17,28,30), we find an excellent agreement of the data with the SM. In other words, we find no signal of new physics beyond the SM in the present precision experiments.

In Fig. 7, we show the result of our global SM fit to all the electroweak data in the (m_t, m_H) plane for three representative α_s values. One can clearly see the positive correlation between the preferred values of m_t and m_H , which is found independently of the assumed α_s value. On the other hand, the preferred range of m_H depends rather sensitively on α_s . For $\alpha_s(m_Z) \lesssim 0.125$, smaller m_H is preferred, whereas for $\alpha_s(m_Z) \gtrsim 0.130$, larger m_H is slightly favored. The m_H dependence of the fit is very mild and no strict

bound on m_H can be given without imposing a constraint on $\alpha_s(m_Z)$. We find e.g. for a relatively small α_s estimate of PDG [68];

$$\chi_{\min}^2 = 7.3 + 0.25[\ln(m_H/24.6)]^2 \quad \text{for } \alpha_s(m_Z) = 0.1134 \pm 0.0035. \quad (37)$$

If we blindly take the above m_H dependence of the fit in the region $60 \text{ GeV} < m_H < \infty$, we find $m_H < 1.0 \text{ TeV}$ (90% C.L.), confirming the trend as observed in refs. [69, 70].

Fig. 7: Electroweak constraints on (m_t, m_H) in the minimal SM, for three selected α_s and at $\delta_\alpha = 0$.

Instead, we may allow the electroweak data alone to constrain $\alpha_s(m_Z)$ as well, extending the analysis of the LEP electroweak working group [32]. We find the following parametrization of our global fit to all the electroweak data (17,28,30) in terms of the three parameters $(m_t, m_H, \alpha_s(m_Z))$ in the minimal SM:

$$m_t = 147 - 3\left(\frac{\alpha_s - 0.12}{0.01}\right) - 5\left(\frac{\delta_\alpha}{0.10}\right) + 12.8\left(\ln \frac{m_H}{100}\right) + 0.9\left(\ln \frac{m_H}{100}\right)^2 \begin{cases} +[15 - 0.7 \ln(m_H/100)] \\ -[17 - 0.9 \ln(m_H/100)] \end{cases} \quad (38a)$$

$$\chi_{\min}^2 = 7.2 + \left(\frac{\alpha_s - 0.117}{0.0067}\right)^2 + (1.77 - 13.0\alpha_s)\left[\ln \frac{m_H}{23.1} - \left(\frac{\alpha_s - 0.101}{0.021}\right)^4\right]^2 + \left(\frac{\delta_\alpha}{0.10}\right)^2. \quad (38b)$$

Here m_t and m_H are measured in GeV units. The parametrization reproduces the correct χ_{\min}^2 within a few % accuracy in the range $0.11 < \alpha_s(m_Z) < 0.13$ and $60 < m_H(\text{GeV}) < 1000$. A rather complicated functional form of χ_{\min}^2 above gives the aforementioned α_s dependence of the preferred m_H range. If we allow m_t to take arbitrary values in the range, $100 < m_t(\text{GeV}) < 200$, then the above fit gives

$$\alpha_s(m_Z)_{\overline{\text{MS}}} = 0.118 + 0.0018 \ln(m_H/100) \pm 0.006. \quad (39)$$

The mean value above is, however, also sensitive to m_t . We may further allow m_t and α_s to be freely fitted by the above electroweak data, and find χ_{\min}^2 for a given m_H :

$$\chi_{\min}^2 = 6.9 + 0.114[\ln(m_H/13.5)]^2 \quad \text{for free } \alpha_s(m_Z). \quad (40)$$

Again in the region $60 \text{ GeV} < m_H < \infty$, this leads to a formal constraint on m_H : $m_H < 3.4 \text{ TeV}$ (90%*C.L.*). The upper bound is, however, clearly outside the region of validity of our perturbative framework.

Finally, we show in Table 2 the complete list of all the input data (except for α , G_F and m_Z) and the corresponding minimal SM predictions for several sets of (m_t, m_H, α_s) values. The total χ^2 of each sector is also shown in the table, which is obtained by properly taking account of the correlations among the errors which are all given in the text. We see clearly from the table that the present electroweak experiments are consistent with the SM, perhaps except for a combination of a heavy top and a light Higgs; see the $(m_t, m_H) = (200, 100)\text{GeV}$ column in the table. Even there, the total χ^2 over the effective number of degrees of freedom, 18, is only 1.2. We find the table very useful, nevertheless, because we can read off from it the significance of the future improvements in the precision experiments.

Table 2

	data	no-EW	IBA	SM					
m_t (GeV)		—	150	150	150	200	200	120	200
m_H (GeV)		—	100	100	1000	100	1000	60	1000
$\alpha_s(m_Z)$		0.120	0.120	0.120	0.120	0.120	0.120	0.110	0.130
S		—	-0.2115	-0.2115	-0.0540	-0.2478	-0.0903	-0.2332	-0.0892
T		—	0.5852	0.5852	0.3002	1.2322	0.9110	0.3144	0.8960
U		—	0.3032	0.3032	0.2984	0.4103	0.4055	0.2136	0.4045
$\bar{\delta}_G$		—	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055	0.0055
$1/\bar{\alpha}(m_Z^2)$		128.85	128.72	128.72	128.72	128.71	128.71	128.73	128.71
$\bar{s}^2(m_Z^2)$		0.2312	0.2312	0.2312	0.2325	0.2295	0.2309	0.2317	0.2309
$\bar{g}^2(m_Z^2)$		0.5486	0.5552	0.5552	0.5540	0.5578	0.5566	0.5539	0.5565
$\bar{\delta}_b(m_Z^2)$		—	—	-0.0079	-0.0079	-0.0123	-0.0123	-0.0058	-0.0122
$\bar{s}^2(0)$		0.2388	0.2396	0.2396	0.2408	0.2380	0.2393	0.2401	0.2393
$\bar{g}^2(0)$		0.5486	0.5480	0.5480	0.5468	0.5506	0.5493	0.5469	0.5492
$\bar{g}_W^2(0)$		0.4218	0.4226	0.4226	0.4208	0.4259	0.4240	0.4212	0.4239
Γ_Z (GeV)	2.489 ± 0.007	2.485	2.514	2.490	2.482	2.503	2.494	2.480	2.499
σ_h^0 (nb)	41.56 ± 0.14	41.47	41.47	41.45	41.46	41.48	41.49	41.49	41.43
R_ℓ	20.763 ± 0.049	20.807	20.807	20.763	20.740	20.745	20.722	20.708	20.789
$A_{\text{FB}}^{0,\ell}$	0.0158 ± 0.0018	0.0167	0.0168	0.0153	0.0132	0.0182	0.0159	0.0144	0.0158
P_τ	-0.139 ± 0.014	-0.149	-0.150	-0.142	-0.132	-0.155	-0.144	-0.137	-0.144
A_{LR}	0.100 ± 0.044	0.149	0.150	0.142	0.132	0.155	0.144	0.137	0.144
R_b	0.2203 ± 0.0027	0.2185	0.2185	0.2165	0.2166	0.2147	0.2147	0.2174	0.2147
$A_{\text{FB}}^{0,b}$	0.099 ± 0.006	0.105	0.105	0.099	0.092	0.109	0.101	0.096	0.101
$A_{\text{FB}}^{b,c}$	0.075 ± 0.015	0.075	0.075	0.071	0.065	0.078	0.072	0.068	0.072
χ^2		5.35	18.01	3.71	8.28	15.89	6.91	6.24	8.73
g_L^2	0.2980 ± 0.0044	0.2954	0.2941	0.2979	0.2958	0.3018	0.2995	0.2963	0.2994
g_R^2	0.0307 ± 0.0047	0.0308	0.0309	0.0296	0.0298	0.0295	0.0297	0.0296	0.0297
δ_L^2	-0.0589 ± 0.0237	-0.0600	-0.0600	-0.0761	-0.0760	-0.0765	-0.0764	-0.0759	-0.0764
δ_R^2	0.0206 ± 0.0160	0.0185	0.0185	0.0177	0.0178	0.0177	0.0178	0.0177	0.0178
χ^2		0.51	1.09	0.89	1.37	1.72	0.95	1.20	0.94
s_{eff}^2	0.233 ± 0.008	0.239	0.239	0.231	0.233	0.231	0.232	0.233	0.232
ρ_{eff}	1.007 ± 0.028	1.000	0.999	1.011	1.009	1.016	1.013	1.009	1.013
χ^2		0.59	0.77	0.09	0.01	0.21	0.07	0.01	0.07
Q_W	-71.04 ± 1.81	-75.49	-75.57	-73.12	-73.22	-73.13	-73.23	-73.09	-73.23
χ^2		6.04	6.27	1.32	1.46	1.34	1.47	1.28	1.46
$2C_{1u} - C_{1d}$	-0.94 ± 0.26	-0.71	-0.71	-0.72	-0.71	-0.72	-0.72	-0.71	-0.72
$2C_{2u} - C_{2d}$	0.66 ± 1.23	-0.08	-0.07	-0.10	-0.09	-0.11	-0.10	-0.09	-0.10
χ^2		1.95	2.15	1.56	1.83	1.21	1.45	1.71	1.46
m_W	80.25 ± 0.24	79.95	80.25	80.25	80.08	80.57	80.38	80.11	80.38
χ^2		1.53	0.00	0.00	0.49	1.74	0.31	0.33	0.28
χ_{tot}^2		15.97	28.29	7.58	13.43	22.10	11.17	10.76	12.94

In Table 2, we also show the results of two approximations: the 'no-EW' column is obtained by dropping all electroweak corrections to the two-point functions ($S = T =$

$U = 0$) and vertex/box corrections ($\bar{\delta}_G = \bar{\delta}_b = \Gamma_i = B_{ij} = 0$), while retaining the QED running of the charge form factors $\bar{e}^2(q^2)$ and $\bar{s}^2(q^2)$ due to light particles (excluding W and t contributions). The 'IBA' column shows the result of the improved Born approximation, where we retain all the gauge boson propagator corrections and hence keep all the four charge form factors exact but drop all vertex/box corrections ($\bar{\delta}_b = \Gamma_i = B_{ij} = 0$), except for $\bar{\delta}_G$ in the μ decay. It is quite surprising to note that the 'no-EW' fit to all the data is almost as good as the full SM fit for a preferred (m_t, m_H) range, and that it is significantly *better* than the 'IBA' fit in which all the most important electroweak corrections are supposed to be contained, including the dominant m_t^2 corrections in the T parameter. Even more strikingly, if we further set $\bar{\delta}_G = 0$ in IBA, which may be called a genuine IBA, we obtain $\bar{s}^2(m_Z^2) = 0.2293$ and the total χ^2 jumps to 71. This confirms the observation of ref. [71] that there is no evidence of the genuine electroweak correction in the present electroweak precision experiments, because of the accidental cancellation between the propagator corrections and the remaining vertex/box corrections. Strictly speaking, we do not yet have a 'clean evidence' [72] of the genuine electroweak radiative effect.

5. CONCLUSIONS

We reported the result of our new global study [6] of the electroweak precision measurements. It introduces four charge form factors $\bar{e}^2(q^2)$, $\bar{s}^2(q^2)$, $\bar{g}_Z^2(q^2)$ and $\bar{g}_W^2(q^2)$ associated with the four gauge boson propagators ($\gamma\gamma$, γZ , ZZ and WW) of the $SU(2)_L \times U(1)_Y$ models. By assuming negligible new physics contributions to vertex and box corrections, we can determine these charge form factors accurately from precision experiments at the one-loop level. Our approach allows us to test the electroweak theory at several qualitatively different levels: first, the $SU(2)_L \times U(1)_Y$ universality can be tested by taking all the four charge form factors to be free parameters; second, the running of the form factors can be tested against the expectations of the SM; and, third, the normalization of the three form factors $\bar{s}^2(m_Z^2)$, $\bar{g}_Z^2(m_Z^2)$, $\bar{g}_W^2(0)$ can be tested against the prediction of the minimal SM. The data show excellent agreement with the SM at all stages of the above tests.

We clearly need further improvements in the precision experiments in order to identify a signal of new physics beyond the SM. We find that the two polarization asymmetries at high energies, P_τ and A_{LR} , are most effective in this regard since they constrain the parameter $\bar{s}^2(m_Z^2)$ directly without suffering from the QCD uncertainty. At low energies, two polarization experiments in the $e-q$ sector, the polarized eD scattering and the APV measurements, may have the potential of identifying physics beyond the $SU(2)_L \times U(1)_Y$ universality. We should note, however, that a better measurement of the hadronic vacuum polarization effect $\delta_\alpha = \delta[(\Delta_\alpha^{-1})_{\text{hadrons}}]$ is needed in order for us to look beyond the SM through the electroweak radiative effects.

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